Period 4, Jan 30, 2025

unit circle
$(\cos\theta, \sin\theta)$

$\boxed{\sin x = \frac{1}{2}.}$ $0 \le x \le 2\pi$

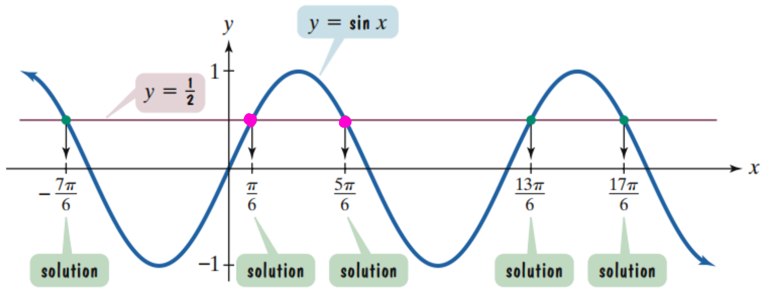
$\sin\frac{\pi}{6} = \frac{1}{2}$

$\sin\frac{5\pi}{6} = \frac{1}{2}$

$\sin 2x = \frac{1}{2} \Rightarrow x = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}$

$\frac{2x}{2} = \frac{\pi}{6\cdot 2}$ $\quad \frac{5\pi}{6\cdot 2} = \frac{2x}{2}$

$\frac{2x}{2} = \frac{13\pi}{6\cdot 2}$ $\quad \frac{17\pi}{6\cdot 2} = \frac{2x}{2}$



$\sin\frac{13\pi}{6}$

$\frac{5\pi}{6} + 2\pi$

$\frac{5\pi}{6} + \frac{12\pi}{6}$

$\frac{13\pi}{6} = \frac{\pi}{6} + 2\pi$

$\frac{13\pi}{6} = \frac{\pi}{6} + \frac{12\pi}{6}$

$\frac{17\pi}{6}$

$$x = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2n\pi,$$

Let $n = 0$. Let $n = 1$.

$$x = \frac{\pi}{6} + 2\cdot 0\pi = \frac{\pi}{6} \qquad x = \frac{5\pi}{6} + 2\cdot 0\pi = \frac{5\pi}{6}$$

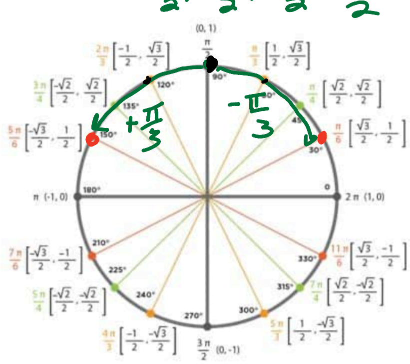
$$x = \frac{\pi}{6} + 2\cdot 1\pi = \frac{\pi}{6} + 2\pi = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6} \qquad x = \frac{5\pi}{6} + 2\cdot 1\pi = \frac{5\pi}{6} + 2\pi = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}.$$

$\frac{\pi}{2} + 2\pi = \frac{\pi}{2} + \frac{4\pi}{2} = \frac{5\pi}{2}$

$\frac{5\pi}{2} + 2\pi = \frac{5\pi}{2} + \frac{4\pi}{2} = \frac{9\pi}{2}$

But wait, there is another way.

$\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$



$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{6} + 2\pi r$

$\theta = \frac{5\pi}{6} + 2\pi r$

Sequence

$\underbrace{\frac{\pi}{2}, \frac{5\pi}{2}}_{+\frac{4\pi}{2}}, \underbrace{\frac{9\pi}{2}}_{\frac{4\pi}{2}}, \underbrace{\frac{13\pi}{2}}_{+\frac{4\pi}{2}} \ldots$

$\frac{\pi}{2} + \left(\frac{4\pi}{2}\right)(n-1)$

$\frac{\pi}{2} + \frac{4\pi}{2}\cdot n - \frac{4\pi}{2}$

$\frac{4\pi n - 3\pi}{2}$

$\theta = \frac{4\pi n - 3\pi}{2} \pm \frac{\pi}{3}$

Solve the equation: $3 \sin x - 2 = 5 \sin x - 1$.

$$x = \frac{7\pi}{6} + 2n\pi \quad \text{and} \quad x = \frac{11\pi}{6} + 2n\pi,$$

around unit circle

$$\cancel{3\sin x} - 2 = 5\sin x - 1$$
$$\cancel{-3\sin x} \qquad -3\sin x$$

$$-2 = 5\sin x - 3\sin x - 1$$

$$-2 = 2\sin x - \cancel{1}$$
$$+1 \qquad + \cancel{1}$$

$$\frac{-1}{2} = \frac{\cancel{2}\sin x}{\cancel{2}}$$

$$-\frac{1}{2} = \sin x$$

$$x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}$$

$5 \sin x = 3 \sin x + \sqrt{3}$.

$$-3\sin x \quad \cancel{-3\sin x}$$

$$\frac{\cancel{2}\sin x}{\cancel{2}} = \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} \quad \text{and} \quad x = \frac{2\pi}{3}$$

$$x = \frac{\pi}{3} + 2\pi n \quad \text{and} \quad x = \frac{2\pi}{3} + 2\pi n$$

or

$$x = \frac{4\pi n - 3\pi}{2} \pm \frac{\pi}{6}$$

tan 3x = 1

$\tan\theta = 1 \Rightarrow \sin\theta = \cos\theta$

$\frac{3x}{3} = \frac{\pi}{3\cdot 4}, \frac{5\pi}{3\cdot 4}, \frac{9\pi}{3\cdot 4}, \frac{13\pi}{3\cdot 4}, \frac{17\pi}{3\cdot 4}, \frac{21\pi}{3\cdot 4}$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}, \frac{25\pi}{4} \ldots$

$x = \underbrace{\frac{\pi}{12}, \frac{5\pi}{12}}_{\frac{4\pi}{12}}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}, \frac{25\pi}{12}$ ← over 2π

$\frac{\pi}{4} + 2\pi = \frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$

$\frac{5\pi}{4} + 2\pi = \frac{5\pi}{4} + \frac{8\pi}{4} = \frac{13\pi}{4}$

$x = \frac{\pi}{12} + \frac{4\pi}{12}(n-1) = \frac{\pi}{12} + \frac{4\pi n}{12} - \frac{4\pi}{12}$

$x = \frac{4\pi n}{12} - \frac{3\pi}{12}$

$\sin\frac{x}{2} = \frac{\sqrt{3}}{2}.$

$\sin\theta = \frac{\sqrt{3}}{2}$

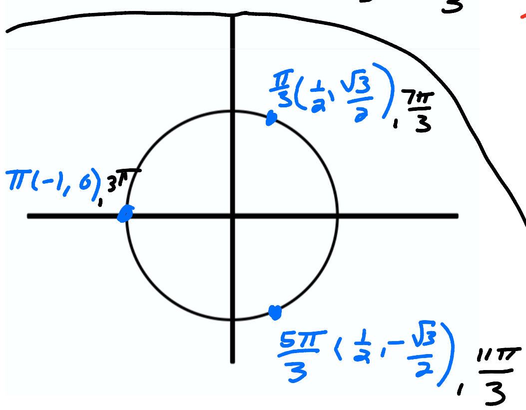
$\cancel{2}\cdot\frac{x}{2} = \left(\frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{13\pi}{3} \ldots\right)2$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

$+\frac{5\pi}{3}$ $\quad + \frac{6\pi}{3}$

$+6\pi/3$ $\quad + \frac{6\pi}{3}$

$2\pi = \frac{6\pi}{3}$



$x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{26\pi}{3} \ldots$

2 cos² x + cos x − 1 = 0

$2\cos^2 x + \cos x - 1 = 2a^2 + a - 1$

$2 \cdot -1 = -2$

$2 \quad -1 = 1$

$2\cos^2 x + 2\cos x - \cos x - 1$

$2\cos x(\cos x + 1) - 1(\cos x + 1)$

$(\cos x + 1)(2\cos x - 1) = 0$

$\cos x + 1 = 0$ or $2\cos x - 1 = 0$

$-1 \quad -1 \qquad\qquad +1 \quad +1$

$\cos x = -1$ or $2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}, \frac{7\pi}{3}, 3\pi, \frac{11\pi}{3}$

$x = \frac{\pi}{3}, \frac{3\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{9\pi}{3}, \frac{11\pi}{3}$

$+\frac{2\pi}{3} \quad +\frac{2\pi}{3} \quad +\frac{2\pi}{3} \ldots$

$\Rightarrow x = \frac{\pi}{3} + \frac{2\pi}{3}(n-1)$

$x = \frac{\pi}{3} + \frac{2\pi n}{3} - \frac{2\pi}{3} \Rightarrow \boxed{x = -\frac{\pi}{3} + \frac{2\pi n}{3}}$

$2\sin^2 x - 3\sin x + 1 = 0.$ $\quad 0 \le x < 2\pi$

$2 \cdot 1 = 2$

$-2 \cdot -1 = -3$

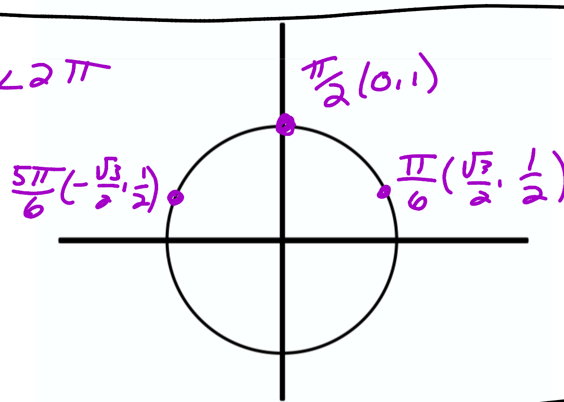
$\underbrace{2\sin^2 x - 2\sin x}_{2\sin x} - \underbrace{\sin x + 1}_{-1}$

$2\sin x(\sin x - 1) - 1(\sin x - 1)$

$(\sin x - 1) = 0$ or $2\sin x - 1 = 0$

$\sin x = 1 \qquad \sin x = \frac{1}{2}$

$\pi/2 \qquad \pi/6, \frac{5\pi}{6}$



$4\sin^2 \frac{\pi}{6} - 1 \qquad 4\sin^2 \frac{5\pi}{6} - 1 \qquad 4\sin^2 \frac{7\pi}{6} - 1$

$4(\frac{1}{2})^2 - 1 \qquad 4(\frac{1}{2})^2 - 1 \qquad 4(-\frac{1}{2})^2 - 1$

$4 \cdot \frac{1}{4} - 1 \qquad 4 \cdot \frac{1}{4} - 1 \qquad 4 \cdot \frac{1}{4} - 1$

$1 - 1 = 0 \qquad 1 - 1 = 0 \qquad 1 - 1 = 0$

$4\sin^2 x - 1 = 0, \quad 0 \le x < 2\pi.$

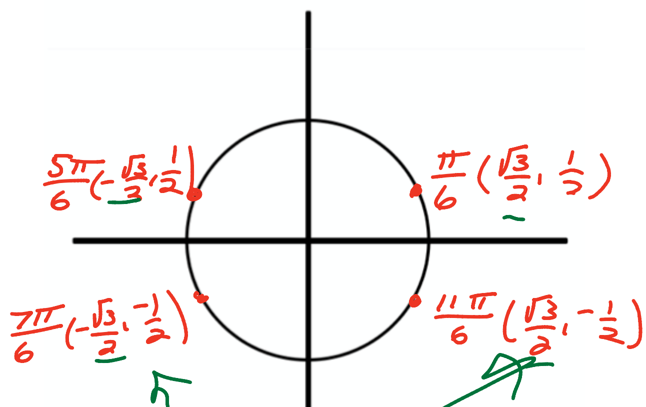
$+1 \quad +1$

$\frac{4\sin^2 x}{4} = \frac{1}{4}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{4}}$

$\sin x = \pm \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

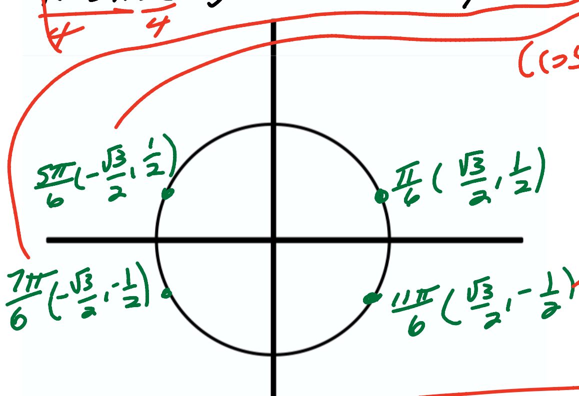


$4\cos^2 x - 3 = 0, \quad 0 \le x < 2\pi.$

$+3 \quad +3$

$\frac{4\cos^2 x}{4} = \frac{3}{4} \Rightarrow \sqrt{\cos^2 x} = \sqrt{\frac{3}{4}} \Rightarrow \cos x = \pm \frac{\sqrt{3}}{2}$

$(\cos\theta, \sin\theta)$



$\tan x \sin^2 x = \cancel{3\tan x}, \quad 0 \le x < 2\pi.$

$-3\tan x \qquad -\cancel{3\tan x}$

$\tan x \sin^2 x - 3\tan x = 0$

$\tan x(\sin^2 x - 3) = 0$

$\tan x = 0 \quad$ or $\quad \sin^2 x - 3 = 0$

$x = 0, \pi, 2\pi \qquad\qquad +3 \quad +3$

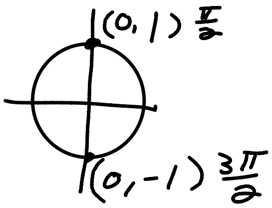
$x = 0, \pi$

$\sqrt{\sin^2 x} = \sqrt{3}$

$\sin x = \pm\sqrt{3} = \pm 1.732$

$-1 \le \sin x \le 1$ impossible

$(\cos\theta, \sin\theta)$



$\sin x = \pm 1.732$

$\sin^{-1} 1.732 = \emptyset$

$2\cos^2 x + 3\sin x = 0, \quad 0 \le x < 2\pi.$

$2(1 - \sin^2 x) + 3\sin x = 0$

$-2\sin^2 x + 3\sin x + 2 = 0 \qquad -2a^2 + 3a + 2$

$-2 \cdot 2 = -4$

$-1 + 4 = 3$

$\underbrace{-2\sin^2 x - \sin x}\ + \underbrace{4\sin x + 2}$

$-\sin x(2\sin x + 1) + 2(2\sin x + 1)$

$(2\sin x + 1)(-\sin x + 2) = 0$

$2\sin x + 1 = 0 \quad$ or $\quad -\sin x + 2 = 0$

$-1 \quad -1 \qquad\qquad +\sin x \quad +\sin x$

$\frac{\cancel{2}\sin x}{\cancel{2}} = \frac{-1}{2} \qquad\qquad 2 = \sin x$

$\sin x = -\frac{1}{2} \qquad\qquad$ Sin is between

$x = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad\qquad -1 \le \sin x \le 1$

no $\emptyset$

$2\sin^2 x - 3\cos x = 0, \quad 0 \le x < 2\pi.$

$2(1-\cos^2 x) - 3\cos x$

$2 - 2\cos^2 x - 3\cos x = 0$

$-2\cos^2 x - 3\cos x + 2 = 0$

$(-2\cos x + 1)(\cos x + 2) = 0$

$-2\cos x + 1 = 0$ or $\cos x + 2 = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos 2x + 3\sin x - 2 = 0, \quad 0 \le x < 2\pi.$

$1 - 2\sin^2 x$

$1 - 2\sin^2 x + 3\sin x - 2 = 0$

$-2\sin^2 x + 3\sin x - 1 = 0$

$-2 \cdot -1 = 2$

$2 + 1 = 3$

$-2\sin^2 x + 2\sin x + 1\sin x - 1 = 0$

$-2\sin x(\sin x - 1) + 1(\sin x - 1)$

$(-2\sin x + 1)(\sin x - 1) = 0$

$-2\sin x + 1 = 0$ or $\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = 1$

$x = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$2 \cdot \sin x \cos x = \frac{1}{2} \cdot 2, \quad 0 \le x < 2\pi.$

$2\sin x \cos x = 1$

$\sin 2x = 1$

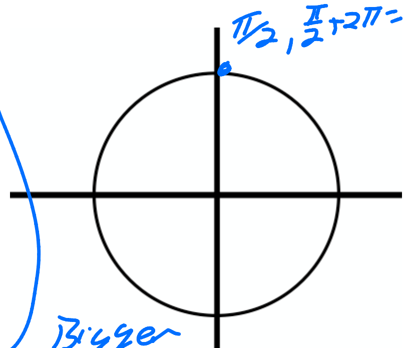
$\frac{\cancel{2}x}{\cancel{2}} = \frac{\pi}{2 \cdot 2}, \frac{5\pi}{2 \cdot 2}, \frac{9\pi}{2 \cdot 2}, \frac{13\pi}{2 \cdot 2}, \ldots$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}, \frac{9\pi}{4} \ldots$

$\frac{9\pi}{4} = 2\frac{1}{4}\pi$

Bigger

$\pi/2, \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}, \frac{9\pi}{2}$



$\sin x - \cos x = 1, \quad 0 \leq x < 2\pi.$

$(\sin x - \cos x) = 1$

Solve each equation, correct to four decimal places, for $0 \leq x < 2\pi$:

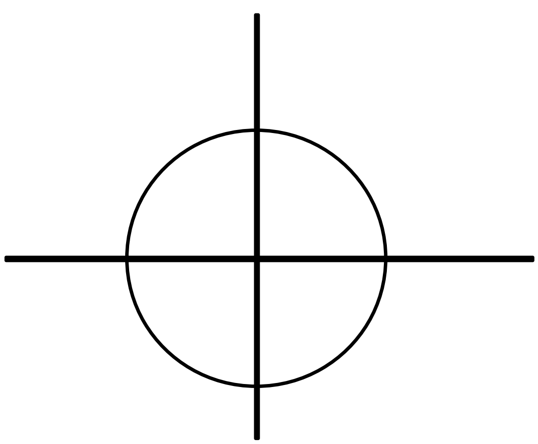
a. $\tan x = 12.8044$ **b.** $\cos x = -0.4317.$

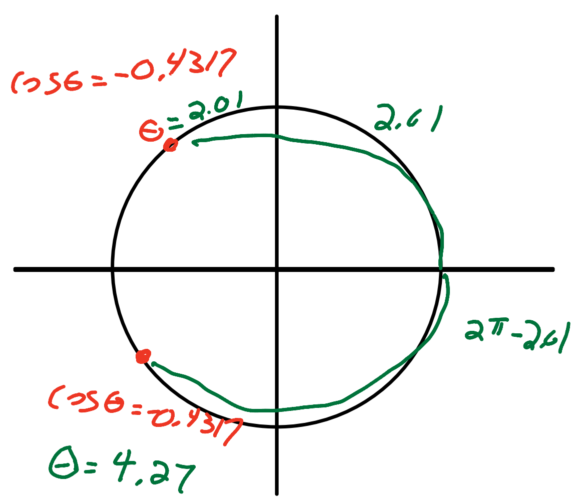
$\cos 4.27 = -0.428$

$\cos 2.01 = -0.425$

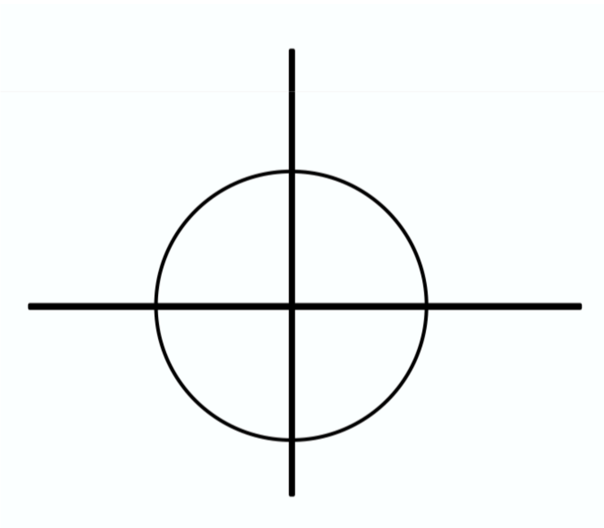
$\cos^{-1}(-0.4317) = 2.01$

$x = 2.01$





b. $\sin x = -0.2315.$



Solve the equation, correct to four decimal places, for $0 \le x < 2\pi$:

$$\sin^2 x - \sin x - 1 = 0.$$

a. $e^{\cos x} = 1$ **b.** $\ln|\sin x| = 0$ **c.** $2^{\tan x} - 8 = 0$

$\ln e^{\cos x} = \ln 1$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

e

$e^0 = |\sin x|$

$1 = |\sin x|$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2 + \sec x - \sec^2 x = 0$ This is an equation containing $\sec x$.

$$\cos^2 x \left(2 + \frac{1}{\cos x} - \frac{1}{\cos^2 x}\right) = 0 \cdot \cos^2 x$$

$$2\cos^2 x + \cos x - 1 = 0$$

Verify the identity.

$\sin(2\theta) - \sin(4\theta) - \sin(6\theta) = -4\cos\theta\sin(2\theta)\cos(3\theta)$

Group the last two terms and factor out a -1 so the first term is positive on the left side of the equation.

$\sin(2\theta) - \sin(4\theta) - \sin(6\theta) = \sin(2\theta) - (\boxed{})$ (Do not simplify.)

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$
$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$\sin 2\theta - [\sin 4\theta + \sin 6\theta]$

$\sin 2\theta - \left[2\sin\frac{4\theta+6\theta}{2}\cos\frac{4\theta-6\theta}{2}\right]$

$\sin 2\theta - \left[2\sin\frac{10\theta}{2}\cos\frac{-2\theta}{2}\right]$

$\sin 2\theta - [2\sin 5\theta\cos(-\theta)]$

$\cos(-\theta) = \cos\theta$

$\sin(-2\theta) = -\sin 2\theta$

$\sin 2\theta - 2\sin 5\theta\cos\theta$

$2\sin\theta\cos\theta - 2\sin 5\theta\cos\theta$

$2\cos\theta[\sin\theta - \sin 5\theta]$

$2\cos\theta\left[2\sin\frac{\theta-5\theta}{2}\cos\frac{\theta+5\theta}{2}\right]$

$2\cos\theta\left[2\sin\frac{-4\theta}{2}\cos\frac{6\theta}{2}\right]$

$2\cos\theta[2\sin(-2\theta)\cos 3\theta]$

$2\cos\theta \cdot -2\sin 2\theta \cdot \cos 3\theta = -4\cos\theta\sin 2\theta\cos 3\theta$

Verify the given sum-to-product formula. Start with the right side and obtain the expression on the left side by using an appropriate product-to-sum formula.

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

What product-to-sum formula can be used to rewrite the right side of the equation?

$x = \frac{\alpha+\beta}{2}$ $\quad y = \frac{\alpha-\beta}{2}$

$$\cos\theta\cos\phi = \frac{1}{2}[\cos(\theta-\phi) + \cos(\theta+\phi)]$$

$\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{1}{2}\left[\cos\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right)\right]$

Rewrite the right side of the equation using an appropriate product-to-sum formula.

$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = 2\cdot\frac{1}{2}\left(\cos\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right)\right)$$

(Do not simplify.)

$\frac{1}{2}[\cos\beta + \cos\alpha]$

The expression determined in the previous step then simplifies to $\cos\alpha + \cos\beta$ using what? Select all that apply.

- ☐ A. Quotient Identity
- ☑ B. Basic arithmatic properties
- ☐ C. Cancellation Property
- ☐ D. Even-Odd Identity
- ☐ E. Pythagorean Identity
- ☐ F. Reciprocal Identity

Product-to-Sum Formulas

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}$

$\frac{\cancel{\alpha}+\beta-\cancel{\alpha}+\beta}{2} = \frac{2\beta}{2} = \beta$

$\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2} = \frac{\alpha+\cancel{\beta}+\alpha-\cancel{\beta}}{2}$

$\frac{2\alpha}{2} = \alpha$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$2\left[\frac{1}{2}(\cos\beta + \cos\alpha)\right]$

$\cos\beta + \cos\alpha$

$$\frac{\sin x - \sin y}{\sin x + \sin y} = \tan\left(\frac{x-y}{2}\right)\cot\left(\frac{x+y}{2}\right)$$

Start with the numerator of the left side and apply the appropriate formula of sum-to-product.

$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Now use the sum-to-product formula on the denominator of the left side.

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

In the numerator and denominator, substitute the expressions found in previous steps. Then divide out the common factor of the expression.

$$\frac{\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{\sin\frac{x+y}{2}\cos\frac{x-y}{2}}$$ (Simplify your answer.)

The fraction from the previous step then simplifies to $\tan\left(\frac{x-y}{2}\right)\cot\left(\frac{x+y}{2}\right)$ using what?

- A. Reciprocal Identity
- B. Quotient Identity
- C. Pythagorean Identity
- D. Even-Odd Identity

$$\frac{\sin x - \sin y}{\sin x + \sin y} = \tan\left(\frac{x-y}{2}\right)\cot\left(\frac{x+y}{2}\right)$$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$$\frac{\cancel{2}\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{\cancel{2}\sin\frac{x+y}{2}\cos\frac{x-y}{2}}$$

$$\frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}} \cdot \frac{\cos\frac{x+y}{2}}{\sin\frac{x+y}{2}} = \tan\frac{x-y}{2} \cdot \cot\frac{x+y}{2}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$
$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Establish the identity.

$$\frac{\sin x - \sin y}{\cos x - \cos y} = -\cot\frac{x+y}{2}$$

Which of the choices shows the key steps that can be used to verify the identity?

A. $$\frac{\sin x - \sin y}{\cos x - \cos y} = \frac{2\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{\cos x - \cos y} = \frac{2\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}} = -\cot\frac{x+y}{2}$$

B. $$\frac{\sin x - \sin y}{\cos x - \cos y} = \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{\cos x - \cos y} = \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}} = -\cot\frac{x+y}{2}$$

C. $$\frac{\sin x - \sin y}{\cos x - \cos y} = \frac{2\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{\cos x - \cos y} = \frac{2\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{-2\cos\frac{x+y}{2}\cos\frac{x-y}{2}} = -\cot\frac{x+y}{2}$$

$$\frac{\sin x - \sin y}{\cos x - \cos y} = \frac{\cancel{2}\,\cancel{\sin\frac{x-y}{2}}\cos\frac{x+y}{2}}{-\cancel{2}\,\cancel{\sin\frac{x-y}{2}}\sin\frac{x+y}{2}} = -\frac{\cos\frac{x+y}{2}}{\sin\frac{x+y}{2}} = -\cot\frac{x+y}{2}$$